Gain properties in a four-level system with a closed interaction contour

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Abstract. We propose a novel scheme for light amplification and gain equalization by quantum interference. We find that the laser amplification can be observed at three specific probe frequencies in a four-level system. Furthermore, we show that three gain peaks can be combined into one to obtain gain equalization.

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1 Introduction

Quantum interference resulting from indistinguishable quantum transition pathways leads to many atomic coherence effects, such as electromagnetically induced transparency (EIT) [1,2], and amplification without inversion [3–6]. The essence of EIT is that atomic coherence is induced in a multilevel system by a strong coupling laser field, which alters the response of the system to a probe laser field. Under the right circumstances, the absorption of a weak probe beam at the resonance frequency can be substantially reduced. The addition of incoherent pumping may lead to amplification with or without inversion, which has been achieved experimentally [3,7,8] in many different schemes.

In usual light amplification, the gain coefficient varies dramatically with the wavelength of the probe laser field. However, this variation is an obstacle in high-speed wavelength-division multiplexing system. Thus equalization of the gain coefficient is very important. In order to achieve gain equalization, Zhang et al. [9] have proposed a method using quantum interference in EDFA, in which the probe laser field passes through three Er^{3+} -doped ZrF4-BaF2-LaF3-AlF3-NaF (ZBLAN) optical fibers with different coherent fields and incoherent pumping. We obtain gain equalization in this paper without any assistant equipment such as the three optical-fibers used by Zhang et al.

We propose a theoretical method to achieve light amplification and gain equalization using the quantum interference effect. We consider a four-level system [10] with a closed interaction contour where the symmetric double-EIT [11] can be observed without incoherent pumping. If an additional incoherent pumping is applied, by numerical calculation, we find that the probe laser is amplified at three different frequencies. Furthermore, with an appropriate incoherent pumping, three gain become degenerate as one to achieve gain equalization. Finally, the separation of the gain frequency of light amplification and the frequency region of gain equalization can be modified by the strengths of coherent laser fields and the incoherent pumping.

This paper is organized as follows: in Section 2, we obtain the gain (absorption) coefficient of the probe laser field by solving the density matrix equation of motion. In Section 3, a few graphic results under specific conditions are illustrated and analyzed. We focus on both the conditions of full-resonance $\delta_1 = \delta_2 = \delta_3 = 0$ and the closed interaction phase $\Phi = \pi/2$. In Section 4, we give a brief summary of the results.

2 The model and the density-matrix equations

The four-level atomic system with a closed interaction contour considered here is shown in Figure 1. Transition $|4\rangle \leftrightarrow |3\rangle$ is driven by a weak probe laser field E_p of frequency ω_p with Rabi frequency $\Omega_p = \mu_{43}E_p/2\hbar$. A strong coupling laser field E_1 of frequency ω_{c1} with Rabi frequency $\Omega_1 = \mu_{13}E_1/2\hbar$ is applied to transition $|1\rangle \leftrightarrow |3\rangle$, and another strong coupling laser field E_2 of

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Fig. 1. A four-level atomic system with a closed interaction contour connected by three coherent laser fields Ω_1 , Ω_m and Ω_2 . An incoherent pump is applied to the probe transition to obtain probe light amplification and gain equalization. The transition driven by the probe laser field Ω_p is outside of the closed interaction contour.

frequency ω_{c2} with Rabi frequency $\Omega_m = \mu_{23} E_2/2\hbar$ drives transition $|2\rangle \leftrightarrow |3\rangle$. Transition $|1\rangle \leftrightarrow |2\rangle$ is coupled by a microwave field E_m of frequency ω_m with Rabi frequency $\Omega_m = \mu_{12} E_2/2\hbar$. In this way, two middle levels $|1\rangle$ and $|2\rangle$ and the upper level $|3\rangle$, connected by a microwave field and two strong coupling fields respectively, form a closed interaction contour. Level $|3\rangle$ is populated by an incoherent pumping process Λ interacting with the transition $|4\rangle \leftrightarrow |3\rangle$. We suppose that level $|1\rangle$ is a metastable state and the transition $|2\rangle \leftrightarrow |1\rangle$ is forbidden for the electricdipole moment. Here μ_{12} is a magnetic-dipole moment and μ_{23} , μ_{13} as well as μ_{43} are electric-dipole moments. Here, μ_{ij} (except μ_{12}) designates the electric-dipole matrix element of transition $|i\rangle \leftrightarrow |j\rangle$ and μ_{12} is a magneticdipole matrix element of transition $|1\rangle \leftrightarrow |2\rangle$. The upper level $|3\rangle$ exhibits spontaneous decay to the lower level $|4\rangle$ and two middle levels $|1\rangle$ and $|2\rangle$ at the same rate Γ_3 .

In the interaction picture, with the rotating-wave approximation and the dipole approximation, the semiclassical interaction Hamiltonian of this four-level atomic system can be expressed as:

$$H_{I} = (\delta_{m} + \delta_{2} - \delta_{1}) |1\rangle \langle 1| + \delta_{m} |2\rangle \langle 2| + (\delta_{m} + \delta_{2}) |3\rangle \langle 3| + (\delta_{m} + \delta_{2} - \delta_{p}) |4\rangle \langle 4| - \Omega_{1} [1\rangle \langle 3|e^{i\varphi_{1}} + |3\rangle \langle 1|e^{-i\varphi_{1}}] - \Omega_{p} [|4\rangle \langle 3|e^{i\varphi_{p}} + |3\rangle \langle 4|e^{-i\varphi_{p}}] - \Omega_{2} [|2\rangle \langle 3|e^{i\varphi_{2}} + |3\rangle \langle 2|e^{-i\varphi_{2}}] - \Omega_{m} [|1\rangle \langle 2|e^{i(\omega t + \varphi_{m})} + |2\rangle \langle 1|e^{-i(\omega t + \varphi_{m})}]$$
(1)

where $\delta_1 = (\omega_3 - \omega_1) - \omega_{c1}$, $\delta_m = (\omega_2 - \omega_1) - \omega_m$, $\delta_2 = (\omega_3 - \omega_2) - \omega_{c2}$ and $\delta_p = (\omega_3 - \omega_4) - \omega_p$ represent the detuning and φ_1 , φ_2 , φ_m and φ_p are the phases corresponding to three coherent laser fields Ω_1 , Ω_2 and Ω_m as well as the weak probe laser field Ω_p , respectively. $\omega = \omega_{c2} + \omega_m - \omega_{c1}$ is defined as the three-photon frequency difference. It is clear that only with the fulfillment of the three-photon resonance condition, i.e. $\omega = 0$, the Hamiltonian operator H_I does not depend on the time and there is a stationary solution to the density matrix equations of motion. So all the discussions which follows is under the condition of three-photon resonance $\omega = 0$. The master equation of motion for the density operator in the interaction picture can be written as

$$\frac{\partial \sigma}{\partial t} = \frac{1}{i\hbar} \left[H_I, \sigma \right] + \Lambda \sigma. \tag{2}$$

For the atomic system under study, by expanding equation (2) in terms of newly defined density matrix elements $\rho_{12} = \sigma_{12}e^{i\varphi_m}$, $\rho_{13} = \sigma_{13}e^{-i\varphi_1}$, $\rho_{14} = \sigma_{14}e^{i(\varphi_p-\varphi_1)}$, $\rho_{23} = \sigma_{23}e^{-i\varphi_2}$, $\rho_{24} = \sigma_{24}e^{i(\varphi_m+\varphi_p-\varphi_1)}$, $\rho_{34} = \sigma_{34}e^{i\varphi_p}$, $\rho_{ii} = \sigma_{ii}$, $\rho_{ij}^* = \rho_{ji}^*$ and $\Phi = \varphi_m + \varphi_2 - \varphi_1$, we can easily obtain the following equations of density matrix

$$\begin{aligned} \frac{\partial \rho_{23}}{\partial t} &= \left[-\gamma_{23} + i\delta_3 \right] \rho_{23} + i\Omega_2 \left[\rho_{33} - \rho_{22} \right] \\ &- i\Omega_1 \rho_{21} e^{-i\Phi} + i\Omega_m \rho_{13} e^{-i\Phi} - i\Omega_p \rho_{24} e^{-i\Phi} \\ \frac{\partial \rho_{13}}{\partial t} &= \left[-\gamma_{13} + i\delta_1 \right] \rho_{13} + i\Omega_1 \left[\rho_{33} - \rho_{11} \right] \\ &+ i\Omega_m \rho_{23} e^{i\Phi} - i\Omega_2 \rho_{12} e^{i\Phi} - i\Omega_p \rho_{14} \\ \frac{\partial \rho_{33}}{\partial t} &= i\Omega_1 \left[\rho_{13} - \rho_{31} \right] + i\Omega_2 \left[\rho_{23} - \rho_{32} \right] \\ &+ i\Omega_p \left[\rho_{43} - \rho_{34} \right] - 3\Gamma_3 \rho_{33} + \Lambda \rho_{44} \\ \frac{\partial \rho_{12}}{\partial t} &= \left[i \left(\delta_1 - \delta_2 \right) \right] \rho_{12} + i\Omega_m \left[\rho_{22} - \rho_{11} \right] \\ &+ i\Omega_1 \rho_{32} e^{-i\Phi} - i\Omega_2 \rho_{13} e^{-i\Phi} \\ \frac{\partial \rho_{24}}{\partial t} &= \left[-\gamma_{24} + i \left(\delta_2 - \delta_p \right) \right] \rho_{24} + i\Omega_m \rho_{14} \\ &+ i\Omega_2 \rho_{34} e^{i\Phi} - i\Omega_p \rho_{23} e^{i\Phi} \\ \frac{\partial \rho_{34}}{\partial t} &= \left[-\gamma_{34} - i\delta_p \right] \rho_{34} + i\Omega_p \left[\rho_{44} - \rho_{33} \right] \\ &+ i\Omega_1 \rho_{14} + i\Omega_2 \rho_{24} e^{-i\Phi} \\ \frac{\partial \rho_{14}}{\partial t} &= \left[-\gamma_{14} + i \left(\delta_1 - \delta_p \right) \right] \rho_{14} + i\Omega_m \rho_{24} \\ &+ i\Omega_1 \rho_{34} - i\Omega_p \rho_{13} \\ \frac{\partial \rho_{22}}{\partial t} &= i\Omega_m \left[\rho_{12} - \rho_{21} \right] + i\Omega_1 \left[\rho_{31} - \rho_{13} \right] + \Gamma_3 \rho_{33} \\ \frac{\partial \rho_{11}}{\partial t} &= i\Omega_m \left[\rho_{21} - \rho_{12} \right] + i\Omega_1 \left[\rho_{31} - \rho_{13} \right] + \Gamma_3 \rho_{33} \\ \rho_{11} + \rho_{22} + \rho_{33} + \rho_{44} = 1 \\ \rho_{ij} = \rho_{ji}^* \end{aligned}$$

here γ_{ij} is the coherent decay rate corresponding to transition $|i\rangle \leftrightarrow |j\rangle$, and in the radiative limit of no dephasing collisions, it can be given by $\gamma_{14} = \gamma_{24} = \Lambda/2$ and $\gamma_{13} = \gamma_{23} = \gamma_{34} = (3\Gamma_3 + \Lambda)/2.$

In this paper, we are mainly interested in the phenomena of symmetric light amplification with three gain peaks and gain equalization. By numerical calculation, we find that the phenomena can be observed just on the basis of realization of symmetric double-EIT. Xue et al. [11] pointed out that the symmetric double-EIT can be obtained only when two conditions are fulfilled: the closed interaction phase $\Phi = \pi/2$ and the full-resonance condition $\delta_1 = \delta_2 = \delta_m = 0$. So in the following calculation, we set $\Phi = \pi/2$ and $\delta_1 = \delta_2 = \delta_m = 0$.



Fig. 2. The gain (absorption) coefficient $Im(\chi)$ as a function of (a) the incoherent pumping Λ and (b) the Rabi frequency Ω when the probe detuning $\delta_p = 0$ is illustrated. The parameters are: (a) $\Omega_1 = \Omega_2 = \Omega_m = 20$ (b) $\Lambda = 20$. Other parameters are: $\Gamma_3 = 1, \, \delta_1 = \delta_2 = \delta_m = 0, \, \Phi = \pi/2.$

probe laser field Ω_p , we find

$$\begin{split} \frac{\rho_{43}^{(1)}}{\Omega_p} &= \frac{\left[\left(-\delta_p - i\gamma_{24} \right) \Omega_1 - i\Omega_2 \Omega_m \right] \rho_{31}^{(0)}}{A} \\ &+ \frac{\left[\left(-\delta_p - i\gamma_{14} \right) \Omega_2 + i\Omega_1 \Omega_m \right] \rho_{32}^{(0)}}{A} \\ &+ \frac{\left[\Omega_m^2 + \gamma_{14}\gamma_{24} - i\delta_p\gamma_{24} - i\delta_p\gamma_{14} - \delta_p^2 \right] \left(\rho_{33}^{(0)} - \rho_{44}^{(0)} \right)}{A} \end{split}$$

$$A = (\delta_p + i\gamma_{14}) (\delta_p + i\gamma_{24}) (\delta_p + i\gamma_{34}) - (\delta_p + i\gamma_{24}) \Omega_1^2 - (\delta_p + i\gamma_{14}) \Omega_2^2 - (\delta_p + i\gamma_{34}) \Omega_m^2.$$
(4)

In the zeroth order for the probe laser field, the coherence created by the coupling fields can be given as:

$$i\rho_{31}^{(0)} = \frac{\Omega_1 \left(\gamma_{12}\Omega_m - \Omega_1 \Omega_2\right) \left(\rho_{33}^{(0)} - \rho_{11}^{(0)}\right)}{\gamma_{12}\gamma_{13}\gamma_{23} + \gamma_{13}\Omega_1^2 + \gamma_{23}\Omega_2^2 + \gamma_{12}\Omega_m^2} \\ + \frac{\Omega_m \left(\gamma_{13}\Omega_1 + \Omega_m \Omega_2\right) \left(\rho_{22}^{(0)} - \rho_{11}^{(0)}\right)}{\gamma_{12}\gamma_{13}\gamma_{23} + \gamma_{13}\Omega_1^2 + \gamma_{23}\Omega_2^2 + \gamma_{12}\Omega_m^2} \\ + \frac{\Omega_2 \left(\gamma_{12}\gamma_{13} + \Omega_2^2\right) \left(\rho_{33}^{(0)} - \rho_{22}^{(0)}\right)}{\gamma_{12}\gamma_{13}\gamma_{23} + \gamma_{13}\Omega_1^2 + \gamma_{23}\Omega_2^2 + \gamma_{12}\Omega_m^2}$$
(5)

$$i\rho_{32}^{(0)} = \frac{\Omega_1 \left(\gamma_{12}\gamma_{23} + \Omega_1^2\right) \left(\rho_{33}^{(0)} - \rho_{11}^{(0)}\right)}{\gamma_{12}\gamma_{13}\gamma_{23} + \gamma_{13}\Omega_1^2 + \gamma_{23}\Omega_2^2 + \gamma_{12}\Omega_m^2} \\ + \frac{\Omega_m \left(\gamma_{23}\Omega_2 - \Omega_m\Omega_1\right) \left(\rho_{22}^{(0)} - \rho_{11}^{(0)}\right)}{\gamma_{12}\gamma_{13}\gamma_{23} + \gamma_{13}\Omega_1^2 + \gamma_{23}\Omega_2^2 + \gamma_{12}\Omega_m^2} \\ - \frac{\Omega_2 \left(\gamma_{12}\Omega_m + \Omega_1\Omega_2\right) \left(\rho_{33}^{(0)} - \rho_{22}^{(0)}\right)}{\gamma_{12}\gamma_{13}\gamma_{23} + \gamma_{13}\Omega_1^2 + \gamma_{23}\Omega_2^2 + \gamma_{12}\Omega_m^2}.$$
(6)

We do not quote the analytical solution of ρ_{ii} (i = 1, 2, 3, 4) as it is a complicated expression. From equations (5) and (6), obviously the real part of the expression

Solving equations (3) for ρ_{34} to the first order of the $\rho_{3i}^{(0)}$ (i = 1, 2) is zero. Then the gain (absorption) coefficient of the probe laser field can be written as:

$$\operatorname{Im}(\chi) = G1 + G2 + G3
G3 = -\frac{N\mu_{43}^2}{2\varepsilon_0} \frac{i\left(c1\delta_p + c2\left(\gamma_{14}\Omega_2 + \Omega_1\Omega_m\right)\right)\rho_{32}^{(0)}}{c2^2 - c1^2}
G2 = -\frac{N\mu_{43}^2}{2\varepsilon_0} \frac{i\left(c1\delta_p + c2\left(\gamma_{24}\Omega_1 + \Omega_m\Omega_2\right)\right)\rho_{31}^{(0)}}{c2^2 - c1^2}
G1 = \frac{N\mu_{43}^2}{2\varepsilon_0}
\times \frac{\left(c1\delta_p\left(\gamma_{14} + \gamma_{24}\right) + c2\left(\Omega_m^2 - \delta_p^2 + \gamma_{14}\gamma_{24}\right)\right)\left(\rho_{33}^{(0)} - \rho_{44}^{(0)}\right)}{c2^2 - c1^2}
(7)$$

where

$$c1 = \delta_p^3 - \gamma_{14}\gamma_{24}\delta_p - (\gamma_{14} + \gamma_{24})\gamma_{34}\delta_p - \delta_p \left(\Omega_1^2 + \Omega_2^2 + \Omega_m^2\right)$$

$$c2 = \gamma_{34} \left(\delta_p^2 - \gamma_{14} \gamma_{24} \right) + \delta_p^2 \left(\gamma_{14} + \gamma_{24} \right) - \left(\gamma_{24} \Omega_1^2 + \gamma_{34} \Omega_m^2 + \gamma_{14} \Omega_2^2 \right).$$

In the following numerical calculation, for simplicity, we set $N\mu_{43}^2/2\varepsilon_0 = 1$ and all the parameters are scaled by Γ_3 .

3 Theoretical analysis and calculation

The gain (absorption) coefficient $\text{Im}(\chi)$ can be obtained numerically from equation (7). In our notation, $\text{Im}(\chi) > 0$ means that the probe laser field is amplified. In the following discussion, we numerically debate the properties of light amplification with three gain peaks and gain equalization.

Let us consider the simple case $\Omega_1 = \Omega_2 = \Omega_m = \Omega$. As with the light amplification, the most important term that we are interested in is the maximum gain and the corresponding condition. Im (χ) as a function of the incoherent pumping Λ (the strength of coupling laser fields Ω) when the probe detuning $\delta_p = 0$ is illustrated in Figure 2a



Fig. 3. Under the condition of $\Omega_1 = \Omega_2 = \Omega_m = 20$, gain coefficient $\text{Im}(\chi)$ vs. the probe detuning δ_p for different values of incoherent pumping Λ is demonstrated. The parameters are: the dashed curve $\Lambda = 4$, the dotted curve $\Lambda = 20$ and the solid curve $\Lambda = 48$. Other parameters are the same as those in Figure 2.

(Fig. 2b). The figures show that gain can be observed with a small value of incoherent pumping Λ or coupling Rabi frequency Ω . We also know that, with the increase of $\Lambda(\Omega)$, the gain goes up to the maximum value rapidly and then decreases and approaches zero (a small value) at the large value of $\Lambda(\Omega)$. This means the maximum gain does not require a large value of incoherent pumping Λ and coupling Rabi frequency Ω . The manifest difference between Figures 2a and 2b is the value of the gain coefficient when $\Lambda(\Omega)$ is large. This can be explained as follows. When $\Lambda \gg \Omega$, conventional light amplification plays an important role in the probe gain. The population is totally pumped to level $|3\rangle$ and then localized in the metastable level $|1\rangle$ and so the coherence between levels $|3\rangle$ and $|4\rangle$ is about zero as shown in Figure 2a. But when $\Omega \gg \Lambda$, the main contribution of probe gain comes from the Raman gain due to the effects of atomic coherence ρ_{31} and ρ_{32} .

In this paper, we are especially interested in the symmetric light amplification and want to know whether the gain equalization can be observed in such a system. The gain coefficient Im (χ) vs. the probe detuning δ_p for different values of Λ when the coupling Rabi frequency $\Omega = 10$ is demonstrated in Figure 3a where the dashed curve, the dotted curve and the solid curve correspond to the conditions of $\Lambda = 0.2\Omega$, Ω and 2.4 Ω , respectively. As shown by the dashed curve, with a small incoherent pumping $\Lambda = 0.2\Omega = 4$, we observe symmetric light amplification with three large gain peaks, which can be explained by the dressed-state mechanism. Under the action of coherent laser fields, the upper level $|3\rangle$ is split into three dressed-state levels $|a\rangle$, $|b\rangle$ and $|c\rangle$ and the corresponding eigenvalues can be obtained as $E_a = \omega_3 + \sqrt{3}\Omega$, $E_b = \omega_3$ and $E_c = \omega_3 - \sqrt{3}\Omega$. Due to the effect of incoherent pumping, the probe transition in three channels are amplified and so we can get three symmetric light amplification and the frequency distance between the two next gain peaks is equal to $\sqrt{3}\Omega$.

With the increase of incoherent pumping Λ the symmetric light amplification still exists but the value of gain peaks decreases as shown by the dotted curve in Figure 3. When the incoherent pumping increases to a



Fig. 4. Gain coefficient $\text{Im}(\chi)$ as a function of the probe detuning δ_p is represented. The parameters are: the dashed curve $\Omega_1 = \Omega_2 = 20, \ \Omega_m = 15$ and $\Lambda = 58$ and the solid curve $\Omega_2 = \Omega_m = 20, \ \Omega_1 = 30$ and $\Lambda = 72$. Other parameters are the same as those in Figure 2.

value $\Lambda = 2.4\Omega = 48$, we find that the symmetric light amplification with three gain peaks degenerates into one with an approximate gain equalization in a large frequency region except a very small peak at the gain center. It is well-known that a good gain equalization is required in optical communication, so we need to eliminate the peak at the gain center though it is small. By numerical calculation, as shown in Figure 4, we find that the small peak at the gain center can be totally removed by either reducing the microwave Rabi frequency (solid curve) or augmenting the coupling Rabi frequency (dashed curve). Also by comparing the two curves, we find that with the increase of coherent Rabi frequency (the solid curve), the frequency region in which gain equalization can be observed is augmented but the magnitude of the flat gain decreases.

From all discussions above, a conclusion can be made: for a given coupling Rabi frequency, with the variation of incoherent pumping Λ , two specific phenomena, symmetric light amplification with three gain peaks and gain equalization, can be observed. If we want to obtain the symmetric light amplification (gain equalization), a small (large) incoherent pumping is required. Furthermore, we are motivated to ask the following question: how can the incoherent pumping and coherent Rabi frequency modify the gain profile to get two distinct physical phenomena?

From equations (7), it is easy to see that the probe gain originates from three terms: G1, G2 and G3, which respectively represent the contributions of convential light amplification $\rho_{33} - \rho_{44}$, Raman gain ρ_{31} and ρ_{32} . To understand the role of incoherent pumping on gain profile more clearly, we plot Gi (i = 1, 2, 3) vs. probe detuning δ_p in Figures 5a and 5b, respectively, corresponding to the parameters $\Omega_1 = \Omega_2 = \Omega_m = 20$ and $\Lambda = 4$ (light amplification) and $\Omega_1 = \Omega_2 = 20$, $\Omega_m = 15$ and $\Lambda = 58$ (gain equalization). With a small Λ as shown in Figure 5a, the effects of the atomic coherence G2 (dashed curve corresponding to 10G2) and G3 (dotted curve corresponding to 10G3) are sufficiently small compared to the contribution from the conventional light amplification G1 (solid curve) that they can be neglected, that is to say, the conventional light amplification plays the most important role in probe light amplification. With the increase of Λ , the



Fig. 5. Gi (i = 1, 2, 3) vs. the probe detuning δ_p for two specific physical mechanisms: probe light amplification and gain equalization, are depicted, in which the solid curve, the dashed curve and the dotted curve correspond to G1, 10G2 and 10G3 respectively. The parameters are: (a) $\Omega_1 = \Omega_2 = \Omega_m = 20$ and $\Lambda = 4$ (b) $\Omega_1 = \Omega_2 = 20$, $\Omega_m = 15$ and $\Lambda = 58$. Other parameters are the same as those in Figure 2.

magnitude of Gi (i = 1, 2, 3) is decreased but the variation of G1 is greater than the variation of G2 and G3. When $\Lambda = 58$, though the magnitude of G1 is still larger than those of G2 and G3, the gain profiles of G1 can already be modified faintly by G2 and G3. In other words, under the common action of conventional light amplification and Raman gain, good gain equalization is obtained but the magnitude and the frequency region of the gain equalization is mainly determined by conventional light amplification.

Now we know that the gain equalization can be obtained on the basis of light amplification. But obviously the light amplification can also be observed when $\Omega_m = 0$. So we propose such a question: under the condition of $\Omega_m = 0$, whether can we get gain equalization by varying the incoherent pumping Λ . If we can, which are the details that make our study more advantageous? By numerical calculation, as shown in Figure 6, we find that the gain equalization can also be observed when $\Omega_m = 0$. But compared with the solid curve in Figure 4, we find that under the condition of $\Omega_m = 0$ to obtain gain equalization a larger incoherent pumping is needed. By comparization, we can also find that, due to the larger incoherent pumping, the amplitude of the flat gain decreases and the frequency region of the flat gain diminishes. Thus we can confirm that: Although the model without the microwave laser field is more easily achieved, the system with a closed interaction contour has more advantages in actual practise.

4 Summary

In summary, we have analyzed the gain properties when an incoherent pumping is applied in a four-level system with a closed interaction contour. We not only observe the symmetric light amplification with three gain peaks but also find a specific phenomenon: gain equalization. Zhang [9] have also proposed a method to achieve gain equalization, but it is obtained with assistant equipment: the probe laser passes through three fibers and each time with different applied laser fields, which is different from the scheme we propose in this paper. We obtain the phenomenon by making use of only a four-level system. We also analyzed



Fig. 6. Under the condition of $\Omega_m = 0$, gain coefficient $\text{Im}(\chi)$ vs. the probe detuning δ_p is shown. The parameters are: $\Omega_1 = \Omega_2 = 20$, $\Lambda = 98$ and $\Phi = 0$. Other parameters are the same as those in Figure 2.

the potential physical mechanism of the symmetric light amplification with three gain peaks and the gain equalization, and show that conventional light amplification is responsible for the probe light amplification but gain equalization need a combined effect of both conventional light amplification and stimulated Raman process. Finally, we point out the advantage of this model by comparison with the system without the microwave laser field.

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